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Three-Form Flux with $\mathcal{N} = 2$ Supersymmetry on $\text{AdS}_5 \times \text{S}^5$

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Abstract

In the context of the AdS/CFT correspondence the general form of a three-form flux perturbation to the $\text{AdS}_5 \times \text{S}^5$ solution in the type IIB supergravity which preserves $\mathcal{N} = 2$ supersymmetry is obtained. The arbitrary holomorphic function appearing in the $\mathcal{N} = 1$ case studied by Graña and Polchinski is restricted to a quadratic function of the coordinates transverse to the D3-branes.

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1. Introduction

It was proposed that the type IIB string theory compactified on $\text{AdS}_5 \times \text{S}^5$ has a dual description by the $\mathcal{N} = 4$ super Yang-Mills theory in the large N limit [1, 2, 3]. This conjecture of the AdS/CFT correspondence has been supported by comparison of spectra, correlation functions and anomalies calculated in both of the supergravity and the Yang-Mills theory. (For a review, see ref. [4].) The AdS/CFT correspondence was also studied in various other spacetime dimensions. At first the correspondence was studied for theories with high supersymmetries such as $\mathcal{N} = 4$. To apply it to more realistic models one has to consider theories with lower supersymmetries.

One of the ways to obtain the AdS/CFT correspondence for lower supersymmetric cases is to modify supergravity solutions by adding a perturbation. In ref. [5] a perturbation of the three-form flux was added to the $\text{AdS}_5 \times \text{S}^5$, which breaks $\mathcal{N} = 4$ to $\mathcal{N} = 1$. This perturbation corresponds to fermion mass terms of the three $\mathcal{N} = 1$ chiral multiplets in the $\mathcal{N} = 4$ super Yang-Mills theory and polarizes D3 branes into 5-branes [6, 7]. Similar constructions of the AdS/CFT correspondence with lower supersymmetries were discussed in refs. [8, 9, 10, 11].

The general form of a three-form flux perturbation to the $\text{AdS}_5 \times \text{S}^5$ solution which preserves $\mathcal{N} = 1$ supersymmetry and satisfies the Bianchi identity and the linearized field equation was obtained in ref. [12]. It contains an arbitrary holomorphic function and an arbitrary harmonic function of the coordinates for the directions transverse to the D3-branes. It was argued that the holomorphic function corresponds to a superpotential in the dual super Yang-Mills theory. When the holomorphic function is quadratic in the transverse coordinates, the three-form flux coincides with that of ref. [5].

The purpose of the present paper is to obtain the general form of a three-form flux perturbation to the $\text{AdS}_5 \times \text{S}^5$ solution which preserves $\mathcal{N} = 2$ supersymmetry. We use the result of the $\mathcal{N} = 1$ case [12] and require further that the second supersymmetry is preserved. We find that the arbitrary holomorphic function in the $\mathcal{N} = 1$ case is restricted to a quadratic function of the transverse coordinates. This is a special form of the perturbation studied in ref. [5], which has one vanishing mass. It would be interesting to study a relation of our result to other works on soft breaking of $\mathcal{N} = 4$ to $\mathcal{N} = 2$ in the Coulomb branch [13, 14, 15]. In order to discuss the corresponding dual field theory and its RG flows we need to find

out an exact solution with non-vanishing three-form flux. In addition, it would be also interesting to discuss the brane representations and massive vacua using S-dual transformations.

2. $\text{AdS}_5 \times \text{S}^5$ solution

The field content of the type IIB supergravity in ten dimensions [16, 17] is a metric g_{MN} , a complex Rarita-Schwinger field ψ_M , a real fourth-rank antisymmetric tensor field with a self-dual field strength F_{MNPQR} , a complex second-rank antisymmetric tensor field with a field strength G_{MNP} , a complex spinor field λ and a complex scalar field $\tau = C + ie^{-\Phi}$. We denote ten-dimensional world indices as $M, N, \dots = 0, 1, \dots, 9$ and local Lorentz indices as $A, B, \dots = 0, 1, \dots, 9$. The fermionic fields satisfy chirality conditions $\bar{\Gamma}_{10D}\psi_M = \psi_M$, $\bar{\Gamma}_{10D}\lambda = -\lambda$, where $\bar{\Gamma}_{10D} = \Gamma^0\Gamma^1\cdots\Gamma^9$ is the ten-dimensional chirality matrix. We choose the ten-dimensional gamma matrices Γ^A to have real components.

The field equations of this theory have a solution with the $\text{AdS}_5 \times \text{S}^5$ metric [18, 19]

$$\begin{aligned} g_{MN}dx^Mdx^N &= Z^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{\frac{1}{2}}\delta_{mn}dx^m dx^n \\ &= Z^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{\frac{1}{2}}dr^2 + R^2d\Omega_5^2, \end{aligned} \quad (1)$$

where $M = (\mu, m)$ ($\mu = 0, 1, 2, 3$; $m = 4, 5, \dots, 9$), $r^2 = x^m x^n \delta_{mn}$, $Z = \frac{R^4}{r^4}$ and $d\Omega_5^2$ is the metric of S^5 . The constant R is a radius of AdS_5 and S^5 . (More generally, Z can be an arbitrary function of x^m which is harmonic except at points where D3-branes exist.) The fifth-rank field strength has non-vanishing components

$$\begin{aligned} F_{\mu\nu\rho\sigma m} &= \frac{1}{\kappa Z^2}\epsilon_{\mu\nu\rho\sigma}\partial_m Z, \\ F_{mnpqr} &= -\frac{Z^{\frac{1}{2}}}{\kappa}\epsilon_{mnpqrs}\partial^s Z, \end{aligned} \quad (2)$$

where κ is a coupling constant.

We are interested in how many supersymmetries are preserved by this solution and by a solution with a perturbation of G_{MNP} discussed later. They are found by studying vanishing of local supertransformations of the fermi fields ψ_M and λ . The

supertransformations of the fermi fields [16, 17] in these backgrounds are

$$\begin{aligned}\delta\psi_M &= \frac{1}{\kappa}D_M\epsilon + \frac{1}{16 \cdot 5!}iF_{P_1\dots P_5}\Gamma^{P_1\dots P_5}\Gamma_M\epsilon + \frac{1}{96}G_{NPQ}\left(\Gamma_M{}^{NPQ} - 9\delta_M^N\Gamma^{PQ}\right)\epsilon^*, \\ \delta\lambda &= \frac{1}{24}G_{MNP}\Gamma^{MNP}\epsilon,\end{aligned}\tag{3}$$

where the transformation parameter ϵ is a complex spinor satisfying the chirality condition $\bar{\Gamma}_{10D}\epsilon = \epsilon$. To study the supertransformations for the above backgrounds it is convenient to represent the ten-dimensional gamma matrices as

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu \otimes \mathbf{1}, \\ \Gamma^m &= \bar{\gamma}_{4D} \otimes \gamma^m,\end{aligned}\tag{4}$$

where γ^μ and γ^m are the SO(3,1) and SO(6) gamma matrices respectively. The chirality matrices are defined as

$$\bar{\gamma}_{4D} = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \bar{\gamma}_{6D} = i\gamma^4\gamma^5\gamma^6\gamma^7\gamma^8\gamma^9,\tag{5}$$

which are related to the ten-dimensional one as $\bar{\Gamma}_{10D} = -\bar{\gamma}_{4D}\bar{\gamma}_{6D}$.

The above $\text{AdS}_5 \times \text{S}^5$ solution without a perturbation has 32 supersymmetries [18, 19]. This can be seen as follows. The supertransformation $\delta\lambda$ automatically vanishes, while the vanishing of $\delta\psi_M$ requires

$$\tilde{D}_M\epsilon = 0,\tag{6}$$

where we have defined

$$\begin{aligned}\tilde{D}_\mu &= \partial_\mu - \frac{1}{8Z}\partial_m Z \gamma_\mu \gamma^m (1 + \bar{\gamma}_{4D}), \\ \tilde{D}_m &= \partial_m - \frac{1}{8Z}\partial_n Z (\delta_m^n \bar{\gamma}_{4D} - \gamma_m{}^n (1 + \bar{\gamma}_{4D})).\end{aligned}\tag{7}$$

For solutions of eq. (6) to exist the integrability condition

$$[\tilde{D}_M, \tilde{D}_N]\epsilon = 0\tag{8}$$

must be satisfied. Using the expression (7) it is easy to show that eq. (8) is satisfied for an arbitrary ϵ . Therefore, all of 32 supersymmetries are preserved [18, 19]. From the four-dimensional field theoretical point of view in the AdS/CFT correspondence

16 of them are Poincaré supersymmetries while other 16 are conformal supersymmetries. Thus, we have $\mathcal{N} = 4$ supersymmetry in four dimensions. More explicitly, the solutions of eq. (6) with the chirality $\bar{\gamma}_{4D} = -1$ have a form

$$\epsilon = Z^{-\frac{1}{8}}\eta, \quad (9)$$

where η is an arbitrary constant spinor with the chirality $\bar{\gamma}_{4D} = -1$. These solutions correspond to Poincaré supersymmetries. The solutions with the chirality $\bar{\gamma}_{4D} = +1$ depend on x^μ and correspond to conformal supersymmetries.

3. Three-form flux with $\mathcal{N} = 2$ supersymmetry

By introducing a perturbation of the three-form flux G_{mnp} the $\mathcal{N} = 4$ supersymmetry of the unperturbed supergravity background is broken to lower \mathcal{N} . In ref. [12] the conditions on G_{mnp} for unbroken $\mathcal{N} = 1$ supersymmetry were studied. The supersymmetry parameter is expanded as $\epsilon = \epsilon_0 + \epsilon_1 + \dots$, where ϵ_0 is a solution of eq. (6) for the unperturbed background and ϵ_1 is the first order correction due to the perturbation. Substituting it into eq. (6) ϵ_1 is determined by ϵ_0 . To proceed it is convenient to define complex coordinates z^i ($i = 1, 2, 3$) from x^m

$$z^1 = \frac{1}{\sqrt{2}}(x^4 + ix^7), \quad z^2 = \frac{1}{\sqrt{2}}(x^5 + ix^8), \quad z^3 = \frac{1}{\sqrt{2}}(x^6 + ix^9). \quad (10)$$

It was required in ref. [12] that one of the four Poincaré supersymmetries $\epsilon_0 = Z^{-\frac{1}{8}}\eta$, where η is a constant spinor satisfying

$$\gamma^{\bar{1}}\eta = \gamma^{\bar{2}}\eta = \gamma^{\bar{3}}\eta = 0, \quad (11)$$

is preserved. Here, \bar{i} denote indices of \bar{z}^i , while i denote those of z^i . Using the expression $\bar{\gamma}_{6D} = (1 - \gamma^1\gamma^{\bar{1}})(1 - \gamma^2\gamma^{\bar{2}})(1 - \gamma^3\gamma^{\bar{3}})$ it is easy to see that this ϵ_0 has the chirality $\bar{\gamma}_{4D} = -\bar{\gamma}_{6D} = -1$ appropriate for the Poincaré supersymmetry. Then, this $\mathcal{N} = 1$ supersymmetry restricts the form of G_{mnp} as [12]

$$\begin{aligned} G_{ijk} &= 0, \\ G_{ij\bar{k}} &= \frac{2}{3}\hat{\epsilon}_{\bar{k}}^{pq}\partial^{-2}\partial_p\partial_{[i}\phi\partial_{j]}\partial_q Z + \hat{\epsilon}_{ij}^{\bar{l}}\partial_{\bar{k}}\partial_{\bar{l}}\psi, \\ G_{i\bar{j}\bar{k}} &= \frac{1}{12}\hat{\epsilon}_{\bar{j}\bar{k}}^l\left(2\partial_i\partial_l\phi Z - \alpha\hat{\epsilon}_{il}^{\bar{k}}\partial_{\bar{k}}Z - 4\partial_{[i}\phi\partial_{l]}\right), \\ G_{\bar{i}\bar{j}\bar{k}} &= \frac{1}{6}\hat{\epsilon}_{\bar{i}\bar{j}\bar{k}}\delta^{\bar{l}}\partial_l\phi\partial_{\bar{l}}Z, \end{aligned} \quad (12)$$

where $\phi(z^1, z^2, z^3)$ is an arbitrary holomorphic function, α is an arbitrary constant and ψ is an arbitrary harmonic function.* In eq. (12) $\hat{\epsilon}_{ij}^{\bar{k}}$ and $\hat{\epsilon}_{\bar{i}\bar{j}}^k$ are totally antisymmetric in their indices and take constant values 0, ± 1 regardless of index positions, and $\partial^2 = 2\delta^{i\bar{i}}\partial_i\partial_{\bar{i}}$ is the Laplacian. The three-form flux (12) also satisfies the Bianchi identity as well as the linearized field equation.

We shall obtain conditions on G_{mnp} for unbroken $\mathcal{N} = 2$ supersymmetry. We require that in addition to $\epsilon_0 = Z^{-\frac{1}{8}}\eta$ the second supersymmetry with the parameter

$$\epsilon_0 = Z^{-\frac{1}{8}}\gamma^1\gamma^2\eta \quad (13)$$

is also preserved. This ϵ_0 satisfies

$$\gamma^1\epsilon_0 = \gamma^2\epsilon_0 = \gamma^3\epsilon_0 = 0 \quad (14)$$

and has the chirality $\bar{\gamma}_{4D} = -1$. Comparing eqs. (11) and (14) it is easy to see that the conditions for the second supersymmetry are obtained from eq. (12) by the replacements

$$1 \leftrightarrow \bar{1}, \quad 2 \leftrightarrow \bar{2}, \quad \alpha \rightarrow \alpha', \quad \phi(z^1, z^2, z^3) \rightarrow \phi'(\bar{z}^1, \bar{z}^2, z^3), \quad \psi \rightarrow \psi' \quad (15)$$

for new α' , ϕ' and ψ' .

We now require that the expression (12) and those with the replacements (15) are compatible each other. Let us first consider G_{123} . From the expression (12) we have $G_{123} = 0$. From the other expression we have $G_{123} = \frac{1}{6}\partial_3^2\phi'Z$, which is obtained from $G_{\bar{1}\bar{2}\bar{3}}$ in eq. (12) by the replacements (15). Thus we obtain a condition

$$G_{123} : \quad \partial_3^2\phi' = 0. \quad (16)$$

Similarly, we obtain conditions

$$\begin{aligned} G_{2\bar{2}1} + G_{3\bar{3}1} : \quad & \partial_2\partial_3\phi' = 0, \\ G_{1\bar{1}2} + G_{3\bar{3}2} : \quad & \partial_1\partial_3\phi' = 0, \\ G_{\bar{1}\bar{2}3} : \quad & \partial_3^2\phi = 0, \\ G_{2\bar{2}\bar{1}} + G_{3\bar{3}\bar{1}} : \quad & \partial_2\partial_3\phi = 0, \\ G_{1\bar{1}\bar{2}} + G_{3\bar{3}\bar{2}} : \quad & \partial_1\partial_3\phi = 0, \\ G_{1\bar{2}\bar{3}} : \quad & \partial_1^2\phi = \partial_2^2\phi', \\ G_{\bar{1}\bar{2}\bar{3}} : \quad & \partial_2^2\phi = \partial_1^2\phi'. \end{aligned} \quad (17)$$

*In ref. [12] the constant α is required to vanish by the Bianchi identity. However, we do not agree with this result and leave α non-vanishing.

The component $G_{1\bar{1}3} + G_{2\bar{2}3}$ vanishes in both of the two expressions and gives no condition. These conditions fix the forms of ϕ and ϕ' as

$$\begin{aligned}\phi &= m_1(z^1)^2 + m_2(z^2)^2 + 2az^1z^2 + b_1z^1 + b_2z^2 + b_3z^3, \\ \phi' &= m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2 + 2a'\bar{z}^1\bar{z}^2 + b'_1\bar{z}^1 + b'_2\bar{z}^2 + b'_3\bar{z}^3,\end{aligned}\quad (18)$$

where m_1, m_2, a, a', b_i and b'_i are arbitrary constants. We further obtain conditions

$$\begin{aligned}G_{1\bar{1}23} : \quad & \partial_1^2\psi = \partial_2^2\psi', \\ G_{1\bar{2}3} : \quad & \partial_2^2\psi = \partial_1^2\psi', \\ G_{31\bar{1}} : \quad & \partial_1\partial_2\psi = -\partial_1\partial_2\psi', \quad a = -a'.\end{aligned}\quad (19)$$

By a linear transformation $z^i \rightarrow U^i_j z^j$ ($i, j = 1, 2$) with a unitary matrix U we can set $a = -a' = 0$.

So far we have not used a particular form of Z . From the equation for $G_{1\bar{1}3}$ we obtain

$$\begin{aligned}G_{1\bar{1}3} : \quad & -\frac{1}{6}\partial_1\partial_2\phi Z + \frac{1}{12}(\alpha\partial_3 Z + 2\partial_1\phi\partial_2 Z - 2\partial_2\phi\partial_1 Z) \\ &= \frac{1}{6}\partial_1\partial_2\phi' Z - \frac{1}{12}(\alpha'\partial_3 Z + 2\partial_1\phi'\partial_2 Z - 2\partial_2\phi'\partial_1 Z).\end{aligned}\quad (20)$$

The equation for $G_{2\bar{2}3}$ gives the same condition. Substituting $Z = \frac{R^4}{r^4}$ and eq. (18) into eq. (20) we find $\alpha' = -\alpha$, $b_1 = b_2 = b'_1 = b'_2 = 0$. The remaining conditions are

$$\begin{aligned}G_{1\bar{1}2} : \quad & \partial_1\partial_3\psi' = -\frac{1}{3}(\alpha z^2 + 2b_3\bar{z}^1)\frac{R^4}{r^6}, \\ G_{33\bar{1}} : \quad & \partial_2\partial_3\psi' = \frac{1}{3}(\alpha z^1 - 2b_3\bar{z}^2)\frac{R^4}{r^6}, \\ G_{1\bar{2}3} : \quad & \partial_3^2\psi' = -\frac{2}{3}b_3z^3\frac{R^4}{r^6}, \\ G_{23\bar{3}} : \quad & \partial_1\partial_3\psi = \frac{1}{3}(\alpha\bar{z}^2 - 2b'_3z^1)\frac{R^4}{r^6}, \\ G_{12\bar{2}} : \quad & \partial_2\partial_3\psi = -\frac{1}{3}(\alpha\bar{z}^1 + 2b'_3z^2)\frac{R^4}{r^6}, \\ G_{12\bar{3}} : \quad & \partial_3^2\psi = -\frac{2}{3}b'_3z^3\frac{R^4}{r^6},\end{aligned}\quad (21)$$

where we have used

$$\partial^{-2}\left(\frac{\bar{z}^i\bar{z}^j}{r^8}\right) = -\frac{1}{12}\frac{\bar{z}^i\bar{z}^j}{r^6}.\quad (22)$$

Comparing the equation obtained by applying ∂_3 to the first equation in eq. (21) and that obtained by applying ∂_1 to the third equation we find $\alpha = 0$. Then, eq. (21) determines ψ , ψ' as

$$\begin{aligned}\psi &= -\frac{b'_3 R^4}{12z^3 r^2} + f(z^1, z^2, z^3)\bar{z}^3 + g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3), \\ \psi' &= -\frac{b_3 R^4}{12z^3 r^2} + f'(\bar{z}^1, \bar{z}^2, z^3)\bar{z}^3 + g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3),\end{aligned}\tag{23}$$

where f , f' , g , g' are arbitrary functions of each variables. Substituting eq. (23) into eq. (19) we obtain $b_3 = b'_3 = 0$ and

$$\partial_1^2 g = \partial_2^2 g', \quad \partial_2^2 g = \partial_1^2 g', \quad \partial_1 \partial_2 g = -\partial_1 \partial_2 g'.\tag{24}$$

The conditions that ψ and ψ' in eq. (23) are harmonic are

$$\begin{aligned}\partial^2 g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f(z^1, z^2, z^3), \\ \partial^2 g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f'(\bar{z}^1, \bar{z}^2, z^3).\end{aligned}\tag{25}$$

The functions f and f' do not appear in G_{mnp} as one can see by substituting eq. (23) into eq. (12). We only need to consider g and g' . Eq. (25) means that $\partial^2 g$ and $\partial^2 g'$ are independent of \bar{z}^1, \bar{z}^2 and z^1, z^2 respectively. These conditions are automatically satisfied when g and g' satisfy eq. (24). The functions g and g' need not be harmonic.

Let us summarize the result. The general form of the three-form flux G_{mnp} which preserves the $\mathcal{N} = 2$ supersymmetry at the first order of the perturbation is given by eq. (12) with $\alpha = 0$, ψ replaced by $g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$ satisfying eq. (24) for some function $g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$, and

$$\phi = m_1(z^1)^2 + m_2(z^2)^2.\tag{26}$$

Thus, ϕ , which is an arbitrary holomorphic function in the $\mathcal{N} = 1$ case [12], is severely restricted to a quadratic function in the $\mathcal{N} = 2$ case. At higher orders of the perturbation [20] we would obtain the condition $m_1 = m_2$ since these parameters correspond to masses of two $\mathcal{N} = 1$ chiral multiplets, which should be combined into an $\mathcal{N} = 2$ hypermultiplet.

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